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MODEL DRAWING STRATEGY: A TOOL TO LINK ABSTRACT WORDS TO REAL LIFE

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ABSTRACT

This study was designed to introduce model drawing strategy to teach mathematical word problems at the upper elementary level. In mathematics education word problems are seen as crucial to link abstract thinking to real life scenarios. "Word-problems are considered to play an important role in educational settings as a context for applying mathematics to 'real world issues" (Oviedo, 2005, p.326). Therefore, with the aim of linking abstract words to the real world the model drawing strategy with Polya's problem solving approach was implemented. It was observed that, initially, students were more likely to be reluctant towards the interpretation and representation of the problem but with the passage of time they were able to accept their own abilities of interpretation and meaningful representation of word problems. My analysis reveals that drawings and illustrations not only help students to portray their thinking but also help the teacher to identify where a particular student's misunderstanding lies. Nevertheless, the effective use of this strategy depends on how teachers employ it in the context of the classroom.

Key words: Word problems; drawing strategy; problem solving; teachers; practices

1. INTRODUCTION

This research aims to employ the strategy of drawing in the teaching and learning of word problems at the upper elementary level. Word problems (WPs), regardless of any grade, are usually considered as a basis for application and integration of mathematics in the real world. They provide opportunities to practice and encounter real life problem situations. Armengol and Badía (1998) assert that "word-problems are considered to play an important role in educational settings as a context for applying mathematics to 'real world issues" (p.326). Also, they serve as a mental manipulative and help students in developing their creative, critical and problem solving abilities which are the crux of mathematics education today. Hence, WPs are invaluable in the teaching of mathematics not only to help master mathematics but also for learners' general development (Toom, 2007) and to pave the way for children's abstract thinking. Therefore, students need to be provided with ample ways to understand the problem statement so that learning practices become more meaningful.

2. LITERATURE REVIEW

In Singapore, students are taught a variety of problem solving strategies in order to enable them to understand mathematical problems. One of the most popular problem solving strategies is the model method (Centre for Research in Pedagogy and Practice, 2008). The model method is also suggested in the mathematics syllabi (Ministry of Education, Singapore, 2001a; 2001b) to enable students develop their ability in mathematical problem solving. Consequently, Singaporean grade eight students performed best in the Trends in International Mathematics and Science Studies (TIMSS) in mathematics (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith, 1996; Mullis, Martin, Gonzalez, & Chrostowski, 2004; Mullis, Martin, Gonzalez, Gregory, Garden, & O'Connor, 2000). On the contrary, in the Pakistani context, drawings and the model method are used only at the primary level. At the elementary level teachers confine their teaching to computation and little or no time is spent on drawing or problem solving (Secada, 1991). Instead, students and teachers prefer to translate problem statements into their numerical form regardless of understanding the meaning behind the problem statements. As a result, students find it difficult to implement their learning in real life contexts. Besides, a large number of students of all ages fail to demonstrate grade-level proficiency in solving story problems (Koedinger & Nathan, 2004; National Assessment of Educational Progress, 1992; Neef, Nelles, Iwata & Page, 2003). For instance, Gorgori´o and Planas (2001) found that Pakistani migrants studying in Northern Spain seemed to be passive and nervous in the class when they were asked to use drawings to solve WPs; rather the students were expecting the teacher to write down the steps of the problem on the board. The reason could be that these learners were not familiar with the strategy of drawing as a potential learning resource in the mathematics classroom.

In the context of Pakistan drawings are usually discouraged particularly at the upper elementary level. Many elementary school teachers are concerned about providing opportunities to students to draw their 'thinking' in that it may prevent students from learning abstract forms which according to these teachers are very important. Hence, teachers' conceptions and practices about teaching mathematical problem solving influence the way they plan instructional techniques. Several explanations have been proffered as to why pupils find WPs difficult to solve. Ineffective instruction has been among these reasons (Boggs, 2005; Depaepe, Erik, Corte, & Verschaffel, 2000; Essen & Hamaker, 1990; Ng, 2002; Tchoshanov, 2006; Thevenot & Oakhill, 2008). In my experience, once students are accustomed to drawing a picture/diagram and labelling it with the correct information, it takes less time and creates less confusion in solving WPs. In addition, drawing a picture or a diagram helps students to see different ways to solve a problem which they might not have thought about in the first place. Furthermore, researchers suggest the use of drawings, sketches and models to understand the problem (Jiang & Chua, 2010; Wetzel, 2008). Drawing permits students to visually relate various types of information given in the problem statement to help them determine which mathematical expressions are useful in solving the problem. According to (Huberty, n.d.), drawing pictures and acting things out with objects are excellent strategies when dealing with WPs. Hence, I believe that drawing provides opportunities to understand how students represent problems and information that are not available. Also, Mendes (2005) asserts that "the use of drawings and numerical writing assists the problem solving process" (p.227). It allows students to act and reflect on mathematical concepts as well as enhance their problem solving skills. Several authors, including Leader and Middleton (2004), Montague and Applegate (1993), Polya (1957), Rickard (2005), and Ridlon (2004) suggest that students should be taught to read and understand the problem, come up with a plan, solve the problem, and then check their answer against the fact in the story problem. Besides it provides opportunities for learners to understand and investigate WPs on their own. Students learn and retain better those content areas which are emphasised by their teachers (Eunsook, 1995). Unless teachers realise and understand the importance of employing alternative strategies in teaching word problems they may continue to perpetuate and reinforce traditional practices. For example, writing down data on the board and solving mathematical word problems for students for the sole purpose of exercise completion. Therefore, corresponding to Rickard's work (2005) a teacher needs to be aware of and appreciate the difference between problem solving and merely doing routine exercises. Moreover, teachers need to take a closer look at how students' knowledge of the model method can be used to help students learn abstract forms (Ng, 2003). Having said that it needs to be emphasised that not all word problems can be depicted by using drawings but most can be (Boggs, 2005).

3. METHODOLOGY

The study is qualitative in nature with the aim of improving a teacher's teaching practice of word problems using the model drawing strategy with Polya's problem solving. As problem solving requires a strategic approach to understand and represent the problem Polya's strategic approach, which consists of understanding the problem, devising a plan, carrying out the plan and looking back (Polya, 1957; 1973) was implemented. In addition, seeing the secondary Singaporean students' performance in the TIMSS studies and considering the model method (Yeap & Kaur, 2001; Yeap, Ferrucci, & Carter, 2002), it was decided to unify the model method for my study with some modification. The purpose of modification was to encourage students' creativity and enable them to illustrate their understanding in a variety of ways. Therefore, emphasis was not only on representing the problem situation and relevant concepts using bars (model method) but students were permitted to use a multiplicity of drawings such as rectangles, strips, circles, etc. (model drawing method).

For this study a collaborative action research approach was considered as a powerful tool as it is undertaken by involving other participants for bringing about change (Burns, 1999). Accordingly, I worked with another teacher which entailed observing each other's teaching, giving critical feedback, planning and evaluating each other's lesson as part of my action research cycles. Likewise, collaborative action research provided us opportunities to implement / modify pedagogical strategies on the basis of findings that resulted in bringing a change in the existing practice of a teacher.

4. FINDINGS

4.1 Pre-intervention:
Understanding teaching and learning practices of word problems

To acquire a more in-depth understanding of the existing situation, I analysed documents, carried out classroom observations and interviews with the teacher as well as the students. During this process I realised that the participant teacher, Zohra[1], was guided by her own teacher's teaching practice where the emphasis was on the rules and procedures. My analysis suggests that Zohra's beliefs and practices about teaching WPs were informed by her past and present experiences. I agree with Eunsook (1995) who notes that "the way teachers accumulate their teacher knowledge and belief is expected by their prior educational experiences including primary, secondary and teacher education as well as their life experiences and current teaching experiences" (p.6). Moreover, Zohra was found to have little knowledge about the use of any strategy for teaching WPs. Regarding the use of any strategy in teaching word problems, Zohra said, "Yes, I adopt one strategy that examples needed to be given. Yes, that's the only strategy I know and I use it" (Interview, January 26, 2010). It signifies that to create students' interest in WPs the teacher uses examples related to the problems. However, the long-term applicability of this technique remains questionable. This is because the teacher may not be able to use examples for every problem in the presence of constraints such as completion of the syllabus and limited time. When I suggested the use of drawing as a strategy for teaching word problems more effectively Zohra seemed worried about using it at the upper elementary level. She confided, "We use drawings in lower classes; I do not know how you are going to do it in grade eight" (Discussion, February 08, 2010). I noticed, therefore, that she was not in favour of using the strategy of drawing as a pedagogical tool for teaching mathematical word problems at grade eight. Perhaps it was because of her perception that drawings were only for the lower classes. Besides, the teacher appeared to be constrained by the pressure of syllabus completion. She mentioned that, "Activities (like drawings) take much time and the main thing is we have to complete the syllabus" (Discussion, February 08, 2010). When I inquired from the teacher regarding the use of activities such as drawing and discussions in her mathematics classroom she emphatically responded: "No, no … no; at this level, we don't use drawing and other activities; they have already done it in lower classes" (Discussion, February 08, 2010). Additionally, while observing and discussing with students I realised that they had not experienced engagement in activities for solving WPs. I felt that this issue also seemed to be a source of some concern to students in terms of their understanding and grasping the concept behind the problem statement and overlooking its relationship with the real world. According to one student, "Miss, we copy down the solutions from the board and there is no need of any activity …" (S1, Field notes, February 2, 2010). Almost all the students were of the view that WPs were boring and they did not enjoy them. I believe that it is because they had not come across motivating and interesting approaches and activities for solving WPs. To offset this, my lesson plans included opportunities for students to articulate their own understanding and solve the problem in various ways i.e. through drawings, by restating the problem and by discussion with peers. Besides, while teaching during the reconnaissance stage, I noticed that students were trying to answer the problem through guess work without focusing on the given problem. As a result, at times, students were digressing from the problem. However, during my teaching I realised that a simple story on a familiar topic captured their interest and geared them towards learning with understanding.

4.2 Intervention:

Acquainting the teacher with MDS

Having collected and analysed the baseline information, I chose to introduce model drawing strategy to teach WPs in order to motivate and enable the teacher and students to realise the importance and place of drawing in the mathematics classroom. Moreover, I planned to encourage Zohra to use this strategy in her teaching. I assumed that she would enhance her teaching repertoire by applying this new knowledge in her everyday classroom practice. My role was to introduce her to model drawing strategy (MDS) by demonstration. I also intended to support Zohra in planning and teaching, observe her teaching and provide feedback. According to the plan, I introduced and demonstrated the use of MDS to solve WPs (refer appendix 'A') in grade eight before planning a lesson with Zohra, the purpose of which was to help her understand what MDS was and how it could help students to illustrate their thinking. Zohra stated, "Identifying information and other steps are very nice and they are helping students" (Discussion, January 26, 2010). However, she considered the 'chocolate problem' very simple for grade eight students. On the contrary, it took the entire period of 40 minutes for the students to come up with the drawing for the given question.

During teaching I observed that the teacher and students became more familiar with the purpose of using the strategy of drawing in the context of solving mathematical word problems but the teacher was still not convinced of its relevance and importance in the secondary classroom. She noted, "Yes … drawing helps but, hmm … actually students were confused due to drawing" (Discussion, February 8, 2010). I assumed that students were unable to illustrate the information as they were not able to understand the problem nor were they able to focus on the relevant information and what was asked in the problem.

Students' and the teacher's practices of MDS

As students were not accustomed to using MDS, transformation of data by themselves was a challenge given their limited experience of the strategy. Moreover, I found that students were not even aware of what to draw

[1] Pseudonym

and how to draw. For instance, after reading the problem some students started drawing the 'king, queen and princess' instead of drawing 'chocolates'. I realised that students needed to sift through the information of the word problem and be able to determine what was important and what needed to be drawn. I realised that MDS could help a teacher to gauge students' misunderstandings about how to set up or solve mathematical word problems. Students were encouraged to follow Polya's (1957) steps before they commenced drawing. As a result, in another problem (refer appendix 'A', 'Food for thought, question 1') they were observed to be re-reading and extracting the data by themselves. I noticed that majority of the students were drawing the ribbon first and then transforming it into mathematical language while others attempted to transform the WP into mathematical language and then moved on to drawing. Students drew different objects to represent 'a ribbon' such as a rectangle and a long strip while there were some other students who did it on a number line. This verifies their ability to represent and think in multiple ways. Moreover, students found it easy to solve a problem using drawings as compared to the numeric. A scenario of students' views regarding another problem (refer appendix 'A', 'Food for thought, question 2') is presented below:

S1: It is easy to do with drawings.
S2: It is not boring.
S3: WP ko dirct solve krna mushikil hota hy (*It is difficult to solve WP directly*).
(Students 1, 2 and 3, Field notes, February, 08, 2010)

The preceding data reveal that students found drawing helpful as compared to solving a problem through direct computation. I reflected that it was because the act of drawing helped students learn to visualise the problem (Boggs, 2005) which resulted in getting the solution quickly by enabling them to link mathematics to a real situation. One of the student's work sample is shown in figure 1.

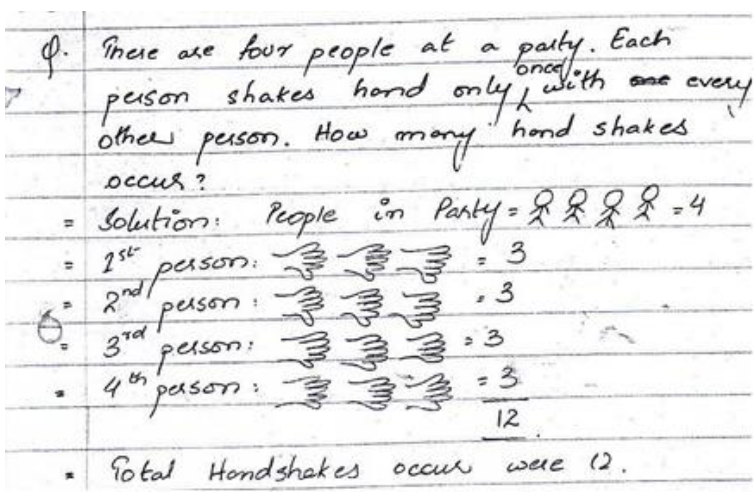

Fig.1. *Sample of a student's work*

Later on, students were shaking their hands to visualise the number of handshakes. Meanwhile, the teacher encouraged them to use drawings: "Also try it by drawing, yes, the main thing is drawing" (Field notes, February 12, 2010). As a result students were able to transform the WP from drawing to displaying the real scenario and then to a numerical form. The teacher was surprised to see the improvement in students' understanding and thinking. To a greater extent this suggests a teacher's and students' acceptance of the strategy. Moreover, the real scenario presented by students indicates students' dynamic improvement regarding the effective use of the strategy. Surprisingly, the teacher seemed to be aware of the limitations of MDS. She felt that MDS could be used in most instances but "If there are 1000 people shaking hands we cannot use drawing" (Reflection, February 13, 2010). This shows her skills of reflecting on the application and limitation of the strategy which I found fascinating. Additionally, her concern about using this strategy to solve every WP indicates that she is not taking the strategy for granted but reflecting on the implications of its use in the classroom.

4.3 Post-intervention:

Acceptance and implementation of MDS by students and the teacher

During observation the teacher was found to read the problem (refer appendix 'B') to students followed by specific instructions to help learners identify the relevant words and numbers by underlining or writing them down on the page (Field notes, February 15, 2010). Further, students were instructed to present the information in the form of a diagram or a picture. Moreover, it was observed that Zohra supported the students who were facing some difficulty in understanding the problem. She suggested, “You can do it in [sic] number of ways” (Observation, February 15, 2010). Moreover, she ensured that students understood the problem statement and the use of MDS. Henjes (2007) states that, “before assessing whether students can use the mathematical skills they have learned by successfully solving a WP, teachers had better make sure that they first understand how to solve one” (p.3). Thus, I realised that the teacher was able to use MDS effectively. For instance, she was able to concurrently focus on Polya’s (1957) steps and the drawing aspect of WPs. It was apparent that the previous classes of teaching and co-teaching had helped Zohra understand the importance of MDS. Furthermore, her reflection revealed that students encountered less difficulty while using drawing as a strategy: “Students did not face any difficulty in understanding the problem; I believe that it is due to allowing them for [sic] drawing”. Further she noted: “Did you noticed [sic] that students were facing difficulty in doing it numerically? (Zohra’s reflection, February 15, 2010). Additionally, I observed the teacher encouraging students to use drawings in her teaching: “I am telling you that drawing can help” (Field notes, February 22, 2010). Some students tried to solve it using bars. Meanwhile, Zohra whispered in my ear, “Now they are on the right track”. She seemed happy about students taking help from MDS. One of the student’s representations of a WP is shown in figure 2.

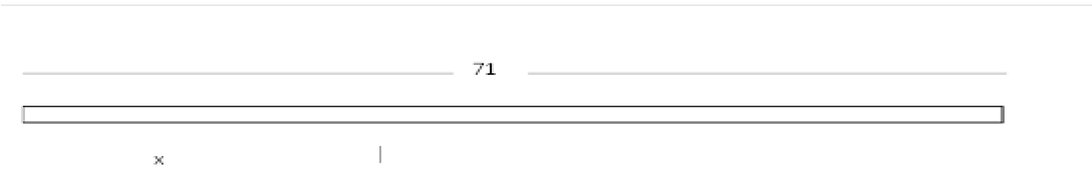

Fig.2. *A student’s representation of a WP*

Further, she explained “because total is 71 so … we will … to get the second part, we will subtract first part” (Field notes, February 22, 2010). Soon after, a number of questions were asked by the students. Some were demanding reasons for writing ‘71– x’ while others considered dealing with word problems as more accessible with MDS. It was realised that students were stuck in writing the equation. When Zohra explained that the larger part exceeded the smaller by 7, one student responded, “We have to subtract ‘71–x’ from ‘x’”. Majority of them agreed with that response and replied: ‘Yes, yes’. One student said, “Miss, now we can do it; give us another word problem”. Besides, a majority of the learners started to solve the question quickly. Data from the reconnaissance stage revealed an overemphasis on words and direct translation by Zohra which hindered students’ understanding of the wordings of mathematical problems. Furthermore, spoon-feeding by the teacher hampered students’ own thinking and reasoning skills as it did not allow them to think deeply about the given question. Therefore, I think students should be given opportunities to explain and demonstrate their thinking skills independently. Hence, on the bases of the post intervention data, it can be said that students were able to use MDS in solving WPs. Moreover, students seemed to make effective use of MDS in solving word problems. Besides, a teacher’s interest and support during co-teaching confirmed her acceptance of the strategy as part of her pedagogical repertoire for teaching WPs in mathematics.

5. DISCUSSION

Overall, observation of Zohra’s teaching with the use of MDS in grade 8 indicated that she did not favour illustrations at that level. Findings show that primarily she was in some ways reluctant to use MDS at upper elementary level. She seemed to believe that MDS was specifically for the lower elementary classes. It seemed that Zohra was convinced of the importance of the strategy as a tool to make links between the abstract and the real. Ng (2003) asserts that teachers can activate students’ prior knowledge of the model method and can use it as a bridge to connect drawings with abstractions. Hence, the preceding data revealed that Zohra was more attuned to the use of MDS and had improved her understanding of implementing it. I believe that, perhaps she was considering the diagram as the sole object of attention in teaching WPs. Otherwise using diagrams can act as scaffolding for the mental screen and help to stabilize images (Mason, 1996). On the contrary, in this study drawing is used in accordance with Polya’s problem solving approach. As a result, students seemed to better articulate their feelings of understanding WPs with joy. Hence, drawing can be used to motivate students to infer meaning from the words and interpret their understanding in the form of illustrations across different levels. It can be a link between the lower and upper elementary levels. Further, as WPs are designed to provoke thinking in learners they cannot always be organised into neat equations; rather WPs can take on any form such as illustrations and drawings. Therefore, Rodrigues (1998) suggests allowing students to use multiple ways of representing their ideas so that they can communicate their thinking with others more effectively. There were evidences of Zohra's enhanced thinking on the use of MDS in her practice of teaching WPs. The important thing is to help students reduce understanding issues in WPs either by abstraction or illustration. Students were moving beyond teacher-led explanation and were trying out different strategies for transforming the data and simplifying the information. Thus, students may now be seen as more successful problem solvers who were constructing a meaningful and rich representation of the problem.

Findings at the reconnaissance stage also revealed that hitherto students had not been given an opportunity to interpret WPs on their own and they believed that WPs could only be interpreted by the teacher. Consequently, instead of forcing my own interpretations, students were encouraged to articulate their own understanding and interpretation of WPs through representation and interpretations. Students' drawing of the 'king, queen and princess' instead of drawing 'chocolates' reflects that learners needed to identify and sort out what was important and what needed to be drawn. On the contrary, "problem representation involves translating a problem from words into a meaningful representation. This could include a carefully constructed arrangement of an idea in one's mind" (Janvier, 1987, p.68). Research in WPs shows that in a majority of cases students' errors are the results of their miscomprehension of the WP (Cummins, Knitsch, Reusser & Weimer, 1988). According to Rodrigues (1998), "the students might be able to recognize what a concept means in the form of a pictorial representation but they may not necessarily have the appropriate vocabulary to express their mathematical thinking" (p.66). Hence, the findings seemed to indicate that drawing a diagram develops the cognitive and metacognitive processes as problem interpretation and representation refer to understanding of concepts along with the linguistic abilities to comprehend the problem.

Furthermore, it is noted that students can typically solve a mathematical problem presented in numbers and equations but when the same problem is presented in the form of a statement or story, many students face difficulty sorting the important information from the distracting one. It is not that they cannot solve the problem – in most cases it is simply that they do not have a plan or strategy for solving the problem (Wetzel, 2008). In the case of WPs students have to first understand the "story" in the problem and then decide what to do with the information provided. There is a lot of extra thinking, reasoning and meaning making. Consequently, students get frustrated while solving the problem (Henjes, 2007). Hence, research in both first and second language contexts indicates that effective learners use appropriate learning strategies, whereas less effective learners use strategies infrequently (Gagne, 1985; O'Malley & Chamot, 1990; Wenden & Rubin, 1987). In mathematics, problem solving strategies have had a positive effect on student achievement (Garderen, 2004; Jiang & Chua, 2010; Pressley & Associates, 1990; Silver & Marshall, 1990). Nevertheless, Chamot, Dale, O'Malley and Spanos (1992) claim that limited research has been conducted on learning strategies in second language contexts.

6. CONCLUSION

In brief, on the bases of a teacher's reflection and my own observation I conclude that the issue of how children interpret and represent WPs is of particular importance to educators because "remediation depends heavily on identifying where a child's misunderstanding lies and what is the nature of the misunderstanding" (Cummins, 1991, p.269). Additionally, teachers need to realise the role played by drawing in transforming data into numerical form as drawing makes transformation accessible and easy for students as compared to learners doing it numerically.

7. RECOMMENDATIONS

Some recommendations are now offered on the bases of the findings of this research:

- Elementary school students and teachers need to develop increased abilities to demonstrate their mathematical understanding in the real world in order to make mathematics more meaningful.
- Students should be encouraged to contextualize word problems as the National Council of Teachers of Mathematics (NCTM, 1989; 1999) assigns a significant role to problem contexts in developing meaning for mathematics.
- Boaler (1994) argued, "mathematics questions should not train students to ignore real world variables but instead enable them to consider and examine the underlying structures and processes which connect classroom questions with real situation[s]" (p.562).
- Efforts should be made to help elementary school teachers recognise the strengths of using MDS and bridge the gap between primary and upper elementary mathematics.
- Further study is also needed to bring reality into the mathematics classroom by creating opportunities for learning and practicing the different aspects of applied problem solving (Verschaffel, 2002).

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APPENDIX A

ACTIVITY SHEET

Name_________________ Date: February 08, 2010

The Chocolate Problem

One night the King couldn't sleep, so he went down into the Royal kitchen, where he found a jar full of chocolate. Being hungry, he took 4 of the chocolates.

Later that same night, the Queen was hungry and couldn't sleep. She, too, found the chocolates and took 3 of what the King had left.

Still later, the first Prince awoke, went to the kitchen, and took 6 of the remaining chocolates; ate only 4 and put the remaining into the jar.

Even later, his brother, the second Prince, ate 5 of what was in the jar.

Finally, the third Prince ate 1 of what was left, leaving only three chocolates for the servants.

How many chocolates were originally in the jar? How many chocolates did the King and the Queen eat altogether? Who ate the most chocolates, and how many?

Clue / Important information

Words

____

____

____

Numbers

____

____

____

Familiar words	unfamiliar words

Translate relevant information into mathematical language

____

____

____

____

Determine mathematical operation needed to solve the problem

____

____

____

____

Draw / illustrate the information

Write down the answer in a complete sentence

____

____

____

Review the problem

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Food for thought

1. A ribbon is 25 m long. Fourteen pieces each of length 14 m are cut from it. The remaining piece is cut into equal lengths of 7 m. How many pieces of length 7 m are there? What is the length of the piece that was left over?

2. There are four people at a party. Each person shakes hands only once with every other person. How many handshakes occur?

APPENDIX B

ACTIVITY SHEET

Name__________________ Date: February 19, 2010

Separate 71 into two equal parts such that one part exceeds the other by 7.

Understand the problem

Read the problem and write down the information.

Words:

____

____

Numbers:

____

____

What is known?

____

____

What are we looking for?

____

____

Restate the problem in your own words:

____

____

Would a diagram or picture help?

Devise a plan

What relationships exist between the known and the unknown?

____

____

Write an equation / expression / formula:

____

____

Carry out the plan

Solve the equation:

Check each step. Do you need to re-evaluate your plan?

Examine the solution

Read the problem again to see how the solution of the equation relates to the question:

Verify the solution with the words of the problem, not your equation: