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ABSTRACT

This paper draws on activity of mathematical concept that elementary mathematics teachers have to prepare and conduct towards the end of their training. A grounded theory approach led to identification of knowledge triplet with three mental activities through which the understanding of mathematical knowledge emerges and could be observed. These are termed as; relationship, approach, and connection among them. This article describes how each of these is characterized and analysis a fragment of one of the model activity performed, showing how each dimension of the triplet can be identified in the given activity. It turns out that triplet can be used as a framework for acquisition of learning mode and develop an understanding of mathematical knowledge for pre-service teachers.

Key words: mathematical concept, knowledge triplet, mental activities, acquisition of learning mode, understanding of mathematical knowledge.

1. INTRODUCTION:

In theoretical perspective the prospective teachers connect new ideas to prior knowledge for understanding learning, it needs an attitude of mind that views learning as an active, socially shared and constructive process by the learners(Shakoor,A., 2010). It is well recognized that students' knowledge is influenced by teachers' knowledge. During the past decades, attention has been given to the teacher's mathematical knowledge. Researchers have also focused on prospective teachers' knowledge. Research on teachers' mathematical knowledge for teaching, Lampert(2001) argued on the ability of teachers to deploy a variety of representations of mathematical knowledge. According to Leinhardt(1990) teachers' initial knowledge is gained from their own school experience and from other personal experiences which can be very influential. While different writers have attempted to categorize this knowledge in different ways. Ball(1990) explored the distinction between what mathematics should be known and how it should be organized. Ma(1999) described the "profound mathematical knowledge" held by Chinese teachers including concepts and procedure. Shulman(1986) developed a new framework for teacher education by introducing the concept of pedagogical content knowledge. Shulman believed that teacher education programs should combine these two knowledge bases to prepare teachers more effectively. In1987 Shulman described seven knowledge bases that identify the teacher's understanding needed to promote comprehension among students but did not mention the aspects of acquiring these knowledge bases that can enhance the teacher's understanding needed to develop comprehension among students and what are the mode of learning for prospective teachers and how they acquire them during their training program.

Mode for learning is an assembling process that prospective teachers learn during their training. For this purpose the task designing should be compulsory during training, to design a task demands the conceptual and procedural understanding of the particular topic. For an example a task on the topic of "surface Area and Volume of Solids" can be shown in hierarchy. This shows that learning mode require a great deal of understanding mathematical knowledge. In the training program prospective teachers should acquire the

knowledge of assembling the topics of content of mathematics, from individual component to the top of the topics. This type of learning demands a lot of efforts, to conceptualize the content of mathematics.

2. UNDERSTANDING MATHEMATICAL KNOWLEDGE:

Understanding Mathematics is a mélange of knowledge which involves the substantive knowledge of mathematics, knowledge of nature & practice of mathematics and the knowledge of connections while a teacher who acquires learning as understanding can develop systematic ways to develop their students' thinking. Teachers should learn mathematical knowledge to connect prior knowledge of mathematics to new concepts by performing the mental activities of the content. Activity based learning can produce supportive learning environment for prospective teachers and their students. We shall discuss here how these three activities are performed and connected with the teachers' learning of mathematical knowledge with understanding.

Understanding of mathematical knowledge can not be defined until and unless engaging it in a specific circular argument, because all complex ideas or processes can be understood at a number of levels and in quite different ways. Some times these are taken as undefined concepts e.g. we can not define a point, it is a basic geometric unit. The development of understanding is a characteristic associated with an individual's knowledge. Therefore we describe understanding as connecting student-teachers prior knowledge of mathematics to new concepts in terms of three mental activity: (a) Constructing Relationship (b) Reshaping approach (c) Connection among relationship & approach. In first dimension of definition, prospective teachers prior knowledge of mathematics is connected to new concepts by these mental activities. This needs to develop understanding of learner's thinking to acquire new knowledge.

3. CONSTRUCTING RELATIONSHIP

Learning with understanding involves steady connections between what teachers already know and the knowledge they are learning now, it will not be sufficient to think of developing understanding simply as appending new concepts and procedures to new existing knowledge. In the long run, developing understanding involves more than simply connecting new knowledge to prior knowledge; it also entails the creation of rich integrated knowledge structures. Similarly examples of elementary algebra of grade-six can be taken for developing algebraic reasoning, e.g. addition of expressions, solution of equations.

4. RESHAPING APPROACH

The second defining characteristic of learning with understanding mathematical knowledge is to reshaping approach the knowledge learned in such a way that clarify its implication. Since learning with understanding is generative, therefore when teachers acquire knowledge with understanding, they can apply the knowledge to learn new topics and solve unfamiliar problems and if they do not understand, they perceive that each topic is an isolated skill they cannot approach the skills, e.g. the algorithmic difference between $2x$ and x^2. As teachers already applied approach to construct relationship so they must understand how to reshape their approach prior to new concepts.

5. CONNECTION AMONG RELATIONSHIP AND APPROACH

Third defining characteristic of learning with understanding mathematical knowledge is connection among relationship & approach i.e. in their learning means teachers consciously examine the knowledge they are acquiring, particularly how it is related to what they already know and to other knowledge they are acquiring. This involves sharing strategies and ideas with developing understanding of connections among the different strategies and ideas needed for the concept or topic to be taught e.g. understanding the algorithm of repeated addition into multiplication as $(x + x + x) = 3x$ where 3 is the co-efficient of x and repeated multiplication into exponent in the simplified form of base and exponent as $(3x)(3x)(3x) = (x + x + x)(x + x + x)(x + x + x) = (x + x + x)^3 = (3x)^3$.

5.1. ACTIVITY TO DEVELOP CONCEPTUAL BASIS FOR ALGEBRAIC GENERALIZATION

Learning for understanding as described earlier, connecting prospective teachers knowledge of mathematics to new concepts in terms of three mental activities:

(a) Constructing Relationship.

(b) Reshaping approach.

(c) Connection among relationship & approach.

An activity is designed to understand how conceptual basis develop understanding for the modes of learning mathematical knowledge. It can be elaborated by a painted cube problem as:

Imagine you have a cube that you dip into paint then

1) How many square surfaces are covered with paint ?
Imagine linking another cube to this one, with two flat ends butted together. Imagine dipping this train of two cubes into paint.

2) How many surfaces are painted this time?
3) Make a train of three cubes (end to end), how many surfaces are painted now?
4) Extend and explore this problem to find ways of predicting how many surfaces are painted for: 10 cubes, 25 cubes and 100 cubes?
Look at the ways in which you represent the data for10 cubes, 25 cubes and 100cubes.

5) Share strategies and ways of thinking about the problem.
6) What are the simplest way of representing the problem?
7) What aspects of this problem are truly algebraic?
8) Reflective; Describe the ways you might be doing algebraic work?

List the ways you assess understanding the concept you teach. In this problem prospective teachers performed these activities for developing generalization as:

6. CONSTRUCTING RELATIONSHIP

Teachers construct the relationship by doing and discussing within the group from question 1 to 4 in the given problem. When teachers try to formulate generalized situations, such as painted cube problem they construct powerful reasoning by counting, recursion whole-object, con textual and rate adjust. They do work like their students work as:

No. of cubes	Painted surfaces
1	6
2	10
3	14
4	18
5	22
6	26
7	30
8	34
9	38
10	42

7. RESHAPING APPROACH

Prospective elementary teachers use reshaping approach to the knowledge learned in ways that clarify how it can be used. In the given problem question no. 5 to 7 teachers apply sharing strategy for the ways of thinking and simplest ways of representing the problem. They generalize that every time when a cube is butted together 4 painted surfaces are increased this aspect of the problem is truly algebraic to form a formula as: painted surfaces = 6 + 4 (n – 1). Where **n** represent the number of cubes.

8. CONNECTION AMONG RELATIONSHIP AND APPROACH

Prospective elementary teachers involve sharing strategies and ideas with developing understanding of connections among the different strategies and ideas needed for the concept or topic to be taught. In the given problem the reflection given to the teachers is the connection among relationship and approach which they discussed with each other or in pairs. Extending the painted cube situations involving the joining of other prisms, such as triangular or pentagonal prisms can enhance teachers learning to further reflect on mathematical knowledge of various strategies.

The focus of recent research into the learning to teach was to enhance prospective teachers' understanding of mathematical knowledge through knowledge triplet, algebra of grade-VI was taken as an example. Secondly, exploring the adequacy of knowledge triplet as a tool for learning mode of acquisition of mathematical knowledge. .Entrekin (1992) reported in a research that teachers may use many knowledge basis to introduce many concepts in mathematics class and may propose ways of integration of the single concept.

9. RESEARCH DESIGN

Qualitative research design was used for exploring the knowledge of elementary algebra by knowledge triplet. The preference was given to qualitative design because the natural setting is the direct source of data (Frankel & Wallen, 2003). In this study the researcher observed participants and collected data in their natural setting. Since this research was intending to enhance prospective teachers' understanding of mathematical concepts for learning to teach, exploring the adequacy of idea of concepts map as a tool and possible uses of it in mathematics education. Creswell (2003) supports this idea. He viewed that this type of study is concerned with the process rather than outcomes or product (p.145)".

Case study as the research method was adopted; this allowed the researcher to explore the tool for learning mode during the acquisition of mathematical knowledge. A case study is particularistic because it focuses on specific phenomenon such as program, event, process, person, institution, or group.

10. RESEARCH SETTING

10.1 SAMPLING AND SAMPLING PROCEDURE

Purposive sampling was adopted. Researcher used Maxwell's (1996) suggestion of using purposeful sampling when persons are "selected deliberately in order to provide important information that [cannot] be gotten as well from other choices (p.70)" as strengthening "A sample in a research study is a group on which information is obtained (Frankel & Wallen 2006, p. 92)". One hundred prospective teachers were participants from the three campuses of university of education district Lahore. The participants were selected randomly among the prospective of following characteristics. They were:

- Enrolled in B.Ed (General)
- Qualified at least 1st semester of B.Ed program
- Mathematics is imperative in their program
- Designed fifteen lessons for teaching, at least five lessons on topics of algebra of sixth-grade.

These boundaries led to follow Maxwell's (1996) suggestion of using purposeful sampling.

11. INSTRUMENTATION

Following three instruments were constructed:

- Questionnaire named Mathematics Teachers' Knowledge Test (MTKT);
- Checklist for classroom observations; and
- Checklist for interviews

12. CONSTRUCTION OF (MTKT)

In constructing the items, techniques of construction and the use of questionnaires were studied. Rimmer (2000) while discussing about opinion and attitude measurement, emphasized the importance of questionnaire and checklists in education research. He said that the educators, especially those involved in guiding the educational activities, were making more and more use of these devices.

For assessing the acquisition of prospective teachers' knowledge MTKT was designed according to the pattern of Test of Teaching knowledge (TTK) of Cambridge University. MTKT was focused on three text book problems of Algebra topics (Algebraic Expression, Co-efficient, base and Exponent) of grade-six. The items of these three problems were constructed to assess the acquisition modes of each aspects of knowledge respectively. .

Each problem consisted of three questions of algebra topics having four choices for each question as shown in see Appendix. A. The assessment of acquired modes of teachers' knowledge was based on: awareness of the Mathematics curricula of grade-six of Punjab Text book board Lahore; algebraic topics, i.e. Concept of constant, Variables, Algebraic expression, Co-efficient, Base and Power.

13. CONSTRUCTION OF CHECKLISTS OF CLASSROOM OBSERVATIONS AND INTERVIEWS

To support MTKT, two checklists, one for classroom observations and the other for interviews were constructed with a two point scale for acquisition modes: appropriate, inappropriate. After the administration of MTKT and collecting the data from the prospective teachers, items of MTKT were classified. This classification was grouped according to the acquired modes of teachers' knowledge. Classification was then categorized for the checklists of classroom observations and interviews. Categories were made to confirm that teaching practice match with their responses on MTKT.

14. VALIDITY AND RELIABILITY

Validity and Reliability of the instruments was determined through experts' opinion and Pilot testing. A pilot study consisting of 30 prospective teachers of University College of Education for Women, Lahore was carried out. On the basis of their pointed out some problems in understanding and suggestions, a few amendments were made in the checklists.

15. DATA COLLECTION

Data were recorded in two phases: in first phase, data was collected with MTKT; and in second phase, data was collected with checklists of classroom observations and interviews with hundred participated prospective teachers. Data were recorded for checklists of classroom observations and interviews with participating prospective teachers. Research observed five prospective teachers per day for twenty minutes in classroom while teaching their selected topic. Each of five teachers was recorded his view on given checklist and interviewed for twenty minutes on same day. All the data from hundred prospective teachers were obtained in four weeks. Classroom observations of one hundred prospective teachers from three campuses of university of education were recorded. After each observation an interview was conducted using the set of interview questions. On the basis of observation criteria of the checklists the special fitting of concepts map as a pedagogical tool for mathematics education was pointed out

with regard to acquisition of mathematical knowledge. Possible uses of the presented concept map together with their advantages and limits were discussed.

16. CONCLUSION

This research has yielded substantiate answers to the questions that derived from the current discussion about acquisition of teachers' mathematical knowledge for teaching Algebra at elementary level. Also on the bases of the results from the analysis of especially designed MTKT, classroom observations and interviews within the limits of this study, it is evident that prospective teachers appropriate acquisition can produce educational balance, effectiveness and enhancement of the subject discipline. So the following conclusions were drawn:

- A balance in learning and teaching modes is essential for understanding and achieving mathematical proficiencies.
- The major goal of teacher education program should be to produce critical thinkers and the active independent learners rather than static learners.
- Teaching is more than dispensing information because learning is more than receiving.
- It is also possible to develop and validate a framework that will establish guidelines for the acquisition of teachers' mathematical knowledge.
- The acquired knowledge of prospective elementary teachers can be assessed by designing the test items, classroom observations and interviews.
- Prospective elementary teachers mathematical knowledge is highly related with the appropriate acquisition of mathematical knowledge they acquire during their training.
- Content and process can be taught simultaneously, what is learned is tied to how one learns.

17. RECOMMENDATIONS

Our society needs the teacher education program to produce such trained prospective who will not only fulfill the manpower requirements but also acquire necessary insight and skill for the future shocks. To keep pace with the social, economic and industrial change the teacher must acquire up dated knowledge of the discipline to become a good communicator and efficient organizer. Hence in the changing scenario the role of teacher must be a more facilitator for students learning.

In the light of changing views about how to learn mathematics a conceptual model of learning mode is presented in the figure below;

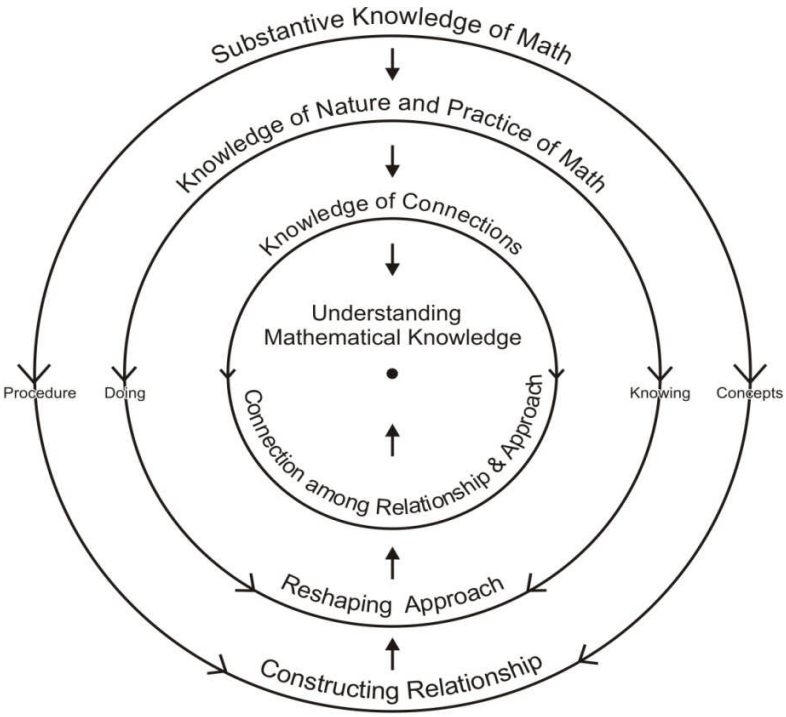

Figure: Conceptual Model of Learning Mode

As the first dimension of definition of acquisition of knowledge by Shakoor, A.(2010) suggests that understanding mathematical knowledge is the pivot for acquiring the aspects of knowledge and conceptual basis for learning modes which is represented in circles. The relationship of these conceptual basis of learning modes can be expressed into the concentric circles of aspects of mathematical knowledge with centre at understanding in the Figure. This relationship is represented in the figure is called the concept ual model of learning modes. The arrows in the circles shows that the modes of learning to acquire the aspects of mathematical knowledge focus on understanding math.

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