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ABSTRACT

The forward kinematics (FK) of serial manipulator robotic arm comprising Double link is calculated by using Newton-Euler equation in framing mathematical modeling. This model helps to determine the concerned mass moment of inertia, velocity and acceleration within specified parameters. The direct kinematics of 3-link pendulum is also stated. The Denavit–Hartenberg parameters (DH) analysis with coordinate transformation function is also enumerated. Thus the concerned results are simulated.

Keyword: forward kinematics (FK), Double link, 3-link pendulum

1. INTRODUCTION

Kinematics is one of the most fundamental disciplines in robotics, providing tools for describing the structure and behavior of robot mechanisms. Robot kinematics refers the analytical study of the motion of a robot manipulator. Formulating the suitable kinematics models for a robot mechanism is very crucial for analyzing the behavior of industrial manipulators. The robot kinematics can be divided into forward kinematics and inverse kinematics. Forward kinematics problem is straightforward and there is no complexity deriving the equations. Hence, there is always a forward kinematics solution of a manipulator. Inverse kinematics is a much more difficult problem than forward kinematics. For the manipulators with more links and whose arms extend into 2-3 dimensions or more the geometry gets much more tedious. The term mechanical system to describe a system or a collection of rigid or flexible bodies may be connected together by joints. Klug et.al has proposed the functionality and the systematic design of a bionic robot arm developed and driven by biologically inspired principles. In this Paper, we will discuss how the motion of a robot mechanism is described, how it responds to its movements, and how the individual pendulum should be coordinated to obtain desired motion at the robot end-effecter. Planar kinematics is more tractable mathematically, compared to general two or three-dimensional kinematics. Here forward kinematics of double link and 3D pendulum robot is discussed with their mathematical modulation and formulation by Newtonian-Euler equation modification.

2. The Newton–Euler equations of motion

The Newton–Euler equations of motion for a rigid body in plane motion are
$m\ddot{r}_C = \Sigma F$ and $I_{Czz}\boldsymbol{\alpha} = \Sigma M_C$,
or using the Cartesian components $m\ddot{x}_C = \Sigma F_x$, $m\ddot{y}_C = \Sigma F_y$, and $I_{Czz}\ddot{\theta} = \Sigma M_C$.
The forces and moments are known and the differential equations are solved for the motion of the rigid body (forward dynamics).

2.1 Double Pendulum

A type of robot having two-link planar chain (double pendulum) is considered, Fig.1a. The proposed virtual model having parameters as links 1 and 2 have the masses $m1$ and $m2$ and the lengths A *to* B = $L1$ =1.25 and BD = $L2$=0.75m. The system is free to move in vertical plane. The local acceleration of gravity is g=*12m/s*. Numerical application: $m1$ =1.5 kg, $m2$ =1.25 kg, Here the equations of motion are to be determined and developed.
The plane of motion is xy plane with the y-axis vertical, with the positive sense directed upward. The origin of the reference frame is at A. The mass centers of the links are designated by
C1(xC1 ; yC1 ; 0) and C2(xC2 ; yC2 ; 0).
The number of degrees of freedom is computed using the relation, $M = 3n - 2c_5 - c_4$,

where 'n' is the number of moving links, c5 is the number of one degree of freedom joints, and c4 is the number of two degrees of freedom joints. For the double pendulum n = 2; c5 = 2; c4 = 0, and the system has two degrees of freedom, M = 2, and two generalized coordinates.
The angles q1(t) and q2(t) are selected as the generalized coordinates as shown in Fig.1.

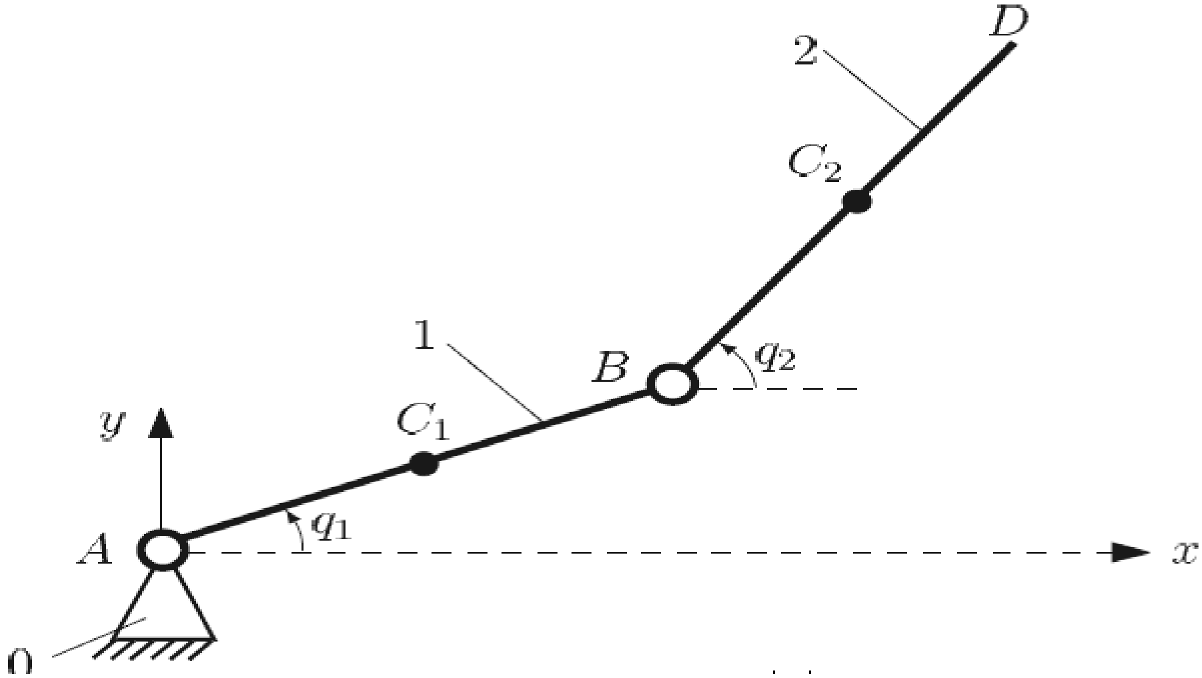

***Figure**-1 Structure of two link (R R) Robot with specified parameters*

2.1(a) Forward Kinematics Double link

The position vector of the center of the mass C1 of the link 1 is $r_{C1} = x_{C1}\, i + y_{C1}\, j$,
Where xC1 and yC1 are the coordinates of C1, $x_{C1} = L_1/2 \cos q_1$ and $y_{C1} = L_1/2 \sin q_1$
The position vector of the center of the mass C2 of the link 2 is $r_{C2} = x_{C2}\, i + y_{C2}\, j$,
Where xC2 and yC2 are the coordinates of C2 described as under:
$x_{C2} = L_1 \cos q_1 + L_2 \cos q_2$ and $y_{C2} = L_1 \sin q_1 + L_2 \sin q_2$,
The velocity vector of C1 is the derivative with respect to time of the position vector of C1 is $v_c = \dot{r}_{c1} = \dot{x}_{c1} I + \dot{y}_{c1} J$,
Where $\dot{x}_{C2} = -L_1 \dot{q}_1 \sin q_1 - L_2/2\, \dot{q}_2 \sin q_2$ and $\dot{y}_{C2} = L_1 \dot{q}_1 \cos q_1 + L_2/2\, q_2 \cos \dot{q}_2$.

The acceleration vector of $C1$ is the double derivative with respect to time of the position vector of C1 is: $a_{C1} = \ddot{r}_{C1} = \ddot{x}_{C1}\, i + \ddot{y}_{C1}\, j$, where $\ddot{x}_{C2} = -L_1 \ddot{q}_1 \sin q_1 - L_1 \dot{q}_1^2 \cos q_1 - L_2/2\, \ddot{q}_2 \sin q_2 - L_2/2\, \dot{q}_2^2 \cos q_2$, $\ddot{y}_{C2} = L_1 \ddot{q}_1 \cos q_1 - L_1 \dot{q}_1^2 \sin q_1 + L_2/2\, \ddot{q}_2 \cos q_2 - L_2/2\, \dot{q}_2^2 \sin q_2$. The angular velocity vectors of the links 1 and 2 are $\boldsymbol{\omega}_1 = \dot{q}_1 k$ and $\boldsymbol{\omega}_2 = \dot{q}_2 k$. The angular acceleration vectors of the links 1 and 2 are $\boldsymbol{\alpha}_1 = \ddot{q}_1 k$ and $\boldsymbol{\alpha}_2 = \ddot{q}_2 k$.

Newton–Euler Equations of Motion for double pendulum are enumerated as under.

The weight forces on the links 1 and 2 are $G_1 = -m_1 g j$ and $G_2 = -m_2 g j$,
The mass moment of inertia of the link 1 with respect to the center of mass $C1$ is $I_{C1} = m_1 L_1^2/12$
The mass moment of inertia of the link 1 with respect to the fixed point of rotation A is $I_A = I_{C1} + m_1 (L_1/2)^2 = m_1 L_1^2/3$
The mass moment of inertia of the link 2 with respect to the center of mass $C2$ is $I_{C2} = m_2 L_2^2/12$
The equations of motion of the pendulum are inferred using the Newton–Euler method. There are two rigid bodies in the system and the Newton–Euler equations are written for each link using the free-body diagrams shown in Fig.1.

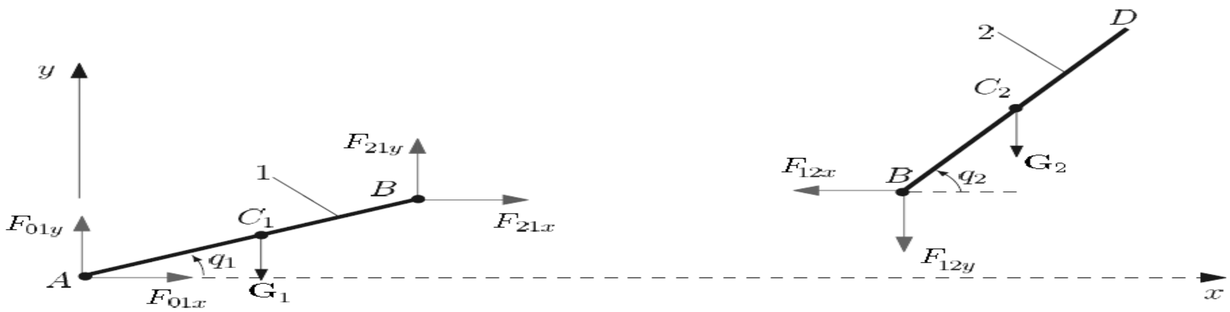

Figure-2 *The dynamic structure of 2D link in each link*

(I) Link 1

The Newton–Euler equations for the link 1 are as extracted from fig-2, $m1aC1$ = F01+F21+G1, *and* $IC1\boldsymbol{\alpha}1$ = r$C1A$×F01+r$C1B$×F21,

where F01 is the joint reaction of the ground 0 on the link 1 at point *A*, and F21 is the joint reaction of the link 2 on the link 1 at point *B* . F01 = *F*01*x*ı+*F*01*y*j and F21 = *F*21*x*i+*F*21*y*j.

Since the link 1 has a fixed point of rotation at *A* the moment sum about the fixed point must be equal to to the product of the link mass moment of inertia about that point and the link angular acceleration. Thus, $IA\boldsymbol{\alpha}1$ = rAC1×G1+rAB×F21, (eq-1)

$$m1L1^2/3\ \ddot{q}1k=\begin{bmatrix} i & j & k \\ xC1 & yC1 & 0 \\ 0 & -m1g & 0 \end{bmatrix} + \begin{bmatrix} i & j & k \\ 0\ +xB & yB & 0 \\ F21x & F21y & 0 \end{bmatrix}$$

$m1L1^2/3\ \ddot{q}1k = (-m1gxC1 + F21yxB - F21xyB)$k.

The equation of motion for link 1 is $m1L2/3\ \ddot{q}1 = (-m1gL12\cos q1 + F21yL1\cos q1 - F21xL1\sin q1)$--------(eq-2)

(II) Link 2

The Newton–Euler equations for the link 2 are $m2aC2$ = F12+G2, (eq-3) and

$IC2\boldsymbol{\alpha}2$ = r$C2B$×F12, ------*(eq-3)*

where F12 = −F21 is the joint reaction of the link 1 on the link 2 at *B*. Equation-3 becomes

$m2\ \ddot{x}C2 = -F21x,\ m2\ \ddot{y}\ C2 = -F21y - m2\ g,$

$$mL2^2\ \ddot{q}2\ k = \begin{bmatrix} \imath & j & k \\ xB-xC2 & yB-yC2 & 0 \\ -F21x & -F21y & 0 \end{bmatrix} \qquad \text{-----(eq-4)}$$

$m2(-L1\ \ddot{q}1\ \sin q1 - L1\ \dot{q}1^2\cos q1 - L2/2\ddot{q}2\ \sin q2 - L2^2\ \dot{q}2\ \cos q2 = -F21x,$ ------*(eq-5)*

$m2(-L1\ \ddot{q}1\ \cos q1 - L1\ \dot{q}1^2\sin q1 - L2/2\ddot{q}2\ \cos q2 - L2^2\ \dot{q}2\ \sin q2 = -F21y - m2g,$ -----*(eq-6)*

$m2L^22/12\ddot{q}2 = L\ 2(-F21y\ \cos q2 + F21x\ \sin q2)$ -------(eq-7)

The reaction components *F*21*x* and *F*21*y* are obtained from Eqs.5 and 6

$F21x = m2(L1\ \ddot{q}1\ \sin q1 + L1\ \dot{q}1^2\cos q1 + L2/2\ddot{q}2\ \sin q2 + L2\ \dot{q}^22\ \cos q2)$,

$F21y = -m2(L1\ \ddot{q}1\ \cos q1 + L1\ \dot{q}1^2\sin q1 + L2/2\ddot{q}2\ \cos q2 + L2\ \dot{q}^22\ \sin q2) + m2g$ -----(eq-8)

The equations of motion are obtained substituting *F*21*x* and *F*21*y* in Eqs.2 and eq-7

$(m2L^21)/3\ \ddot{q}1 = m1gL1/2\ \cos q - m2(L1\ \ddot{q}1\ \cos q1 - L1\ \dot{q}1^2\sin q1 - L2/2\ddot{q}2\ \cos q2 - L2\ \dot{q}^22\ \sin q2 - g)\ L1\cos q1 - m2(L1\ \ddot{q}1\ \sin q1 + L1\ \dot{q}1^2\cos q1 + L2/2\ddot{q}2\ \sin q2 + L2\ \dot{q}^22\ \cos q2)L1\sin.q1$

$(m2L^22)/12\ \ \ddot{q}2 = (m2L2)/2\ (L1\ \ddot{q}1\ \cos q1 - L1\ \dot{q}1^2\sin q1 - L2/2\ddot{q}2\ \cos q2 - L2\ \dot{q}^22\ \sin q2 - g)\ \cos q2 +$

$m2L2/2\ (L1\ \ddot{q}1\ \sin q1 + L1\ \dot{q}1^2\cos q1 + L2/2\ddot{q}2\ \sin q2 + L2\ \dot{q}^22\ \cos q2)\ \sin.q1$ -----(eq-9)

The equations of motion represent two non-linear differential equations. The initial conditions (Cauchy problem) are necessary to solve the equations. At *t* =0 the initial conditions are

$q1(0) = q10,\quad \dot{q}1(0) = \omega 10,$

$q2(0) = q20,\quad \dot{q}2(0) = \omega 20.$

2.1(b) Forward Kinematics 3-link robot

While there may not be any three degree of freedom (d.o.f.) industrial robots with this geometry, the planar 3*R* geometry can be found in many robot manipulators. we can describe the position of the end effector using the coordinates of the reference point (*x*, *y*) and the orientation using the angle f. The 3 end effector coordinates (*x*, *y*, f) completely specify the position and orientation of the end effector in fig-3.

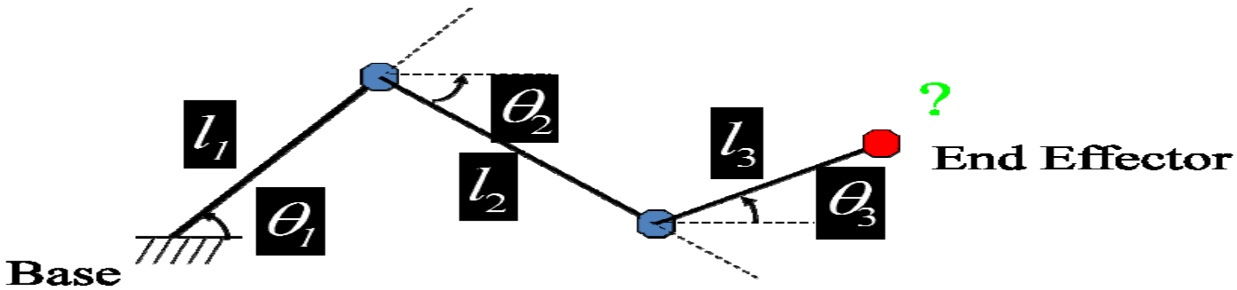

Figure-3 *The model structure of 3-link manipulator*

In above fig-3, the length of first pendulum is L1=2,the length of second pendulum is L2=3, while the length of 3rd pendulum is L3=5,whereas theta1=30, theta2=60,theta3=90, the geometry of above fig-3 can be solved by as under.
x=L1*cos(theta1)+L2*cos(theta1+theta2)+L3*cos(theta1+theta2+theta3) -----(eq-10)
y=L1*sin(theta1)+L2*sin(theta1+theta2)+L3*sin(theta1+theta2+theta3) ------(eq-11)
phi=(theta1+theta2+theta3) ------- (eq-12)

3. Denavit-Hartenberg Transformation matrix

The Denavit–Hartenberg parameters (also called DH parameters) are the four parameters associated with a particular convention for attaching reference frames to the links of a spatial kinematic chain or robot manipulator. The results are obtained by the transformation matrix and associated table.

$$T_i(\theta_i) = \begin{pmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

Table-1 *Denavit–Hartenberg parameters for three pendululm*

DH parameters of the three-link arm *Link*	b_i	θ_i	a_i	α_i
1	0	θ_1 (JV)	a_1	0
2	0	θ_2 (JV)	a_2	0
3	0	θ_3 (JV)	a_3	0

4. Mathematical and Simulation Results

The mathematical model for double pendulum can be calculated through given parameters by MATLAB,
Here the weight force matrix of link-1 and link-2 are G1 =[0 -18 0], G2 =[0 -15 0],
While the moment inertia of link-1 C1 mass is IC1 = 0.1953, and mass moment of inertia to A is
IA = 0.7813 where mass moment inertia of link-2 C2 mass is IC2 = 0.0586.
In triple pendulum, if the θ1= 30 deg, θ2= 60 deg, θ3=90 deg in eq-10 to 12, we get
x = -4.0280
y = -3.2998,
ϕ (phi) = 180

For applying the values θ1= 30 deg, θ2= 60 deg, θ3=90 deg, in the D-H Transformation matrix.
We get

$$T01 = \begin{bmatrix} 0.866 & -0.5000 & 0 & 0 \\ 0.500 & 0.866 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix} \quad T12 = \begin{bmatrix} -0.866 & -0.500 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0.500 & -0.866 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix} \quad T23 = \begin{bmatrix} 0 & -1 & 0 & 0.75 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

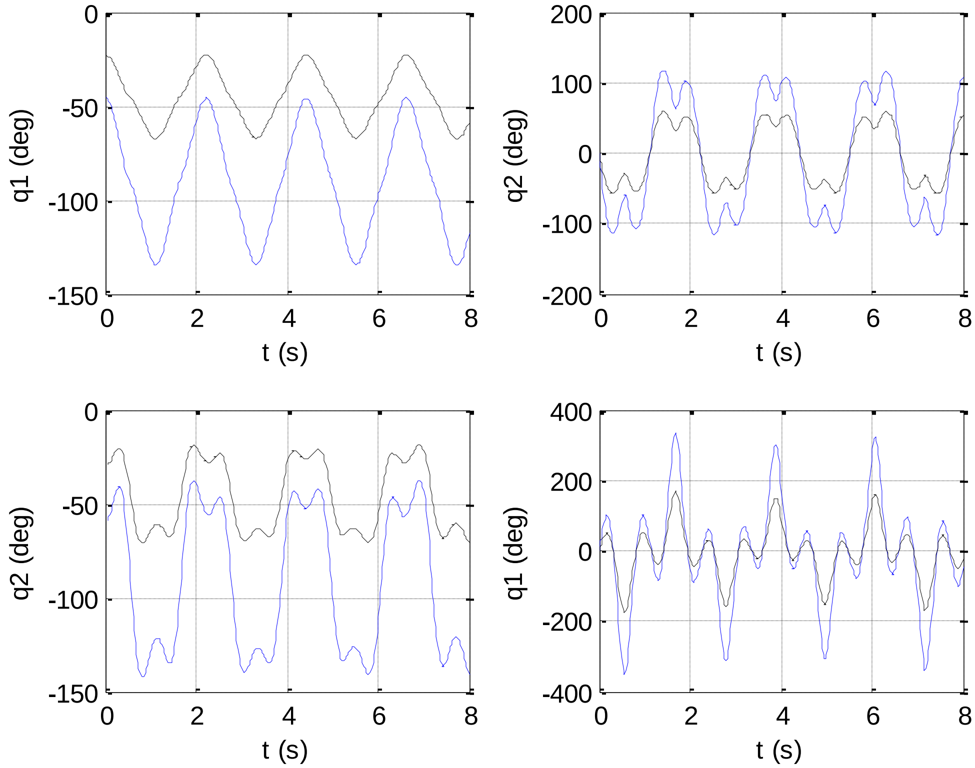

Figure-4 *The Simulation results of two angles of double link robot*

In the 1st quadrant of above figure-4, the angle 'q1' in (deg) represents the orientation of the link-1, here 'blue' shows the original formation of the angle at link-1 while 'black' denotes the half of it. Both correspond to each other. While in the 2nd quadrant the blue line shows the original orientation displayed by another angle 'q2' in 'deg' with link-2, whereas as black as half of it.
Similarly, in 3rd quadrant, the differential of angle 'q2' is shown by blue line and black as half of it displayed by 'black line.

Likewise, in the 4th quadrant, the differential of angle 'q1'in 'deg' is shown by blue line and the black line represents the half of its original orientation.

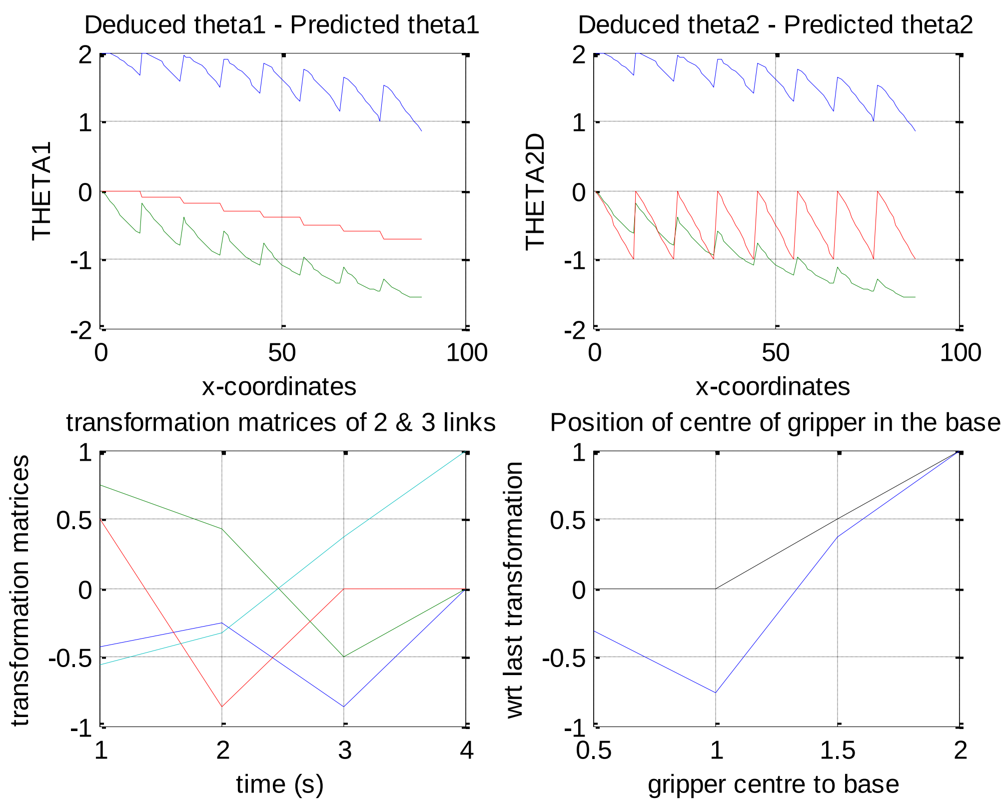

Figure -5 *Orientation of two angles and transformation position 2R pendulum*

In the 1st quadrant of the figure-5, 'Theta-1' is displayed by the deduced and predicted with actual with respect to x-coordinates.
While in the 2nd quadrant of the figure-5, 'Theta-2' is displayed by the deduced and predicted with actual with respect to x-coordinates.
In the 3rd quadrant of fig-5, the transformations of orientation for the double pendulum and tertiary link pendulum to the concerned coordinates with respect to time have been displayed.
In the 4th quadrant of the figure-5, the position of the end-effector to the base frame is stated.
Here gripper centre to the base frame is shown in black color with x-axis, while the y-plane shows transformation analysis referred to the last transformation 'T23'in blue line.

5. Objectives

- This paper helps in building complicated mathematical models
- This provides base for matlab simulation for controlling models
- It helps even in modeling redundant , serial and parallel robots dynamics.

6. Conclusion

In above analysis, the geometry, mathematical modeling of the 'Double (2-link)' pendulum and 3 link planar pendulum robots have been discussed and experimented with specified parameters. Here, the 'Newton–Euler equations' has been applied 2 link robot for calculating the concerned motion equations. Later on these were used determine the mass moment of inertia, velocity and acceleration with respect to the link-1 and link-2. Thus framed angles are calculated and simulated through graphs. The forward kinematics of 3D link pendulum is also calculated. D.H parameters are applied to determine the coordinate transformation matrices through their different orientations and transformation. The simulated results are also plotted. Thus various concerned graphics corresponding to stated analysis and math have been plotted.

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